OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS — 2639

Mechanics 3

Thursday **16 JUNE 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

© OCR 2005 [Y/100/4386] Registered Charity Number: 1066969

1 A ball of mass 0.15 kg bounces on a smooth horizontal surface. Immediately before the bounce the ball has velocity 12 m s^{-1} at an angle of 30° to the surface. The coefficient of restitution between the ball and the surface is 0.7.

(i) Find the vertical component of the velocity of the ball immediately after the bounce. [2]

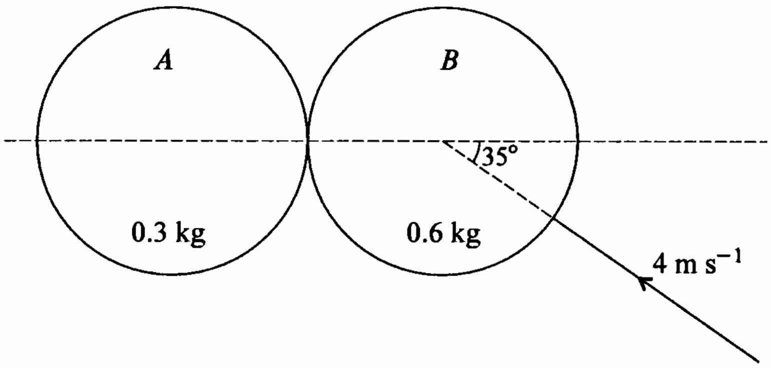
(ii) Hence find the magnitude of the impulse acting on the ball during the bounce. [2]

2 A heavy particle P is attached to one end of a light inextensible string of length 2.45 m. The other end of the string is attached to a fixed point, and P oscillates as a simple pendulum in a vertical plane, with the string making a maximum angle of 0.08 radians on each side of the vertical. Air resistance may be neglected.

(i) Show that the motion is approximately simple harmonic, and find the period. [4]

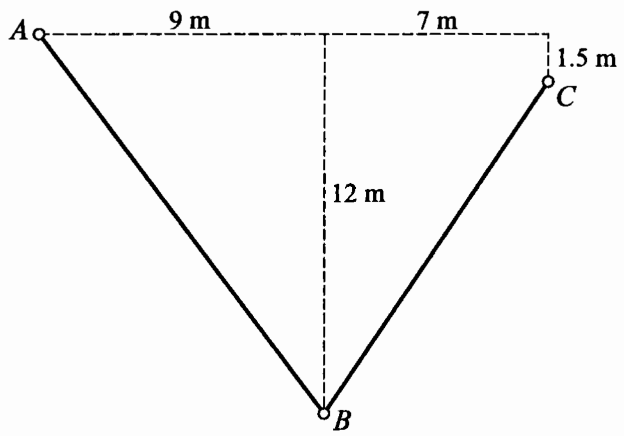
(ii) Find the angle which the string makes with the vertical 1.2 seconds after P has passed through the lowest point of its path. [3]

3



A smooth sphere A, of mass 0.3 kg, is at rest on a horizontal surface. A second smooth sphere B, of mass 0.6 kg and with the same radius as A, is moving on the surface and collides with A. Immediately before the collision, the velocity of B is 4 m s^{-1} at an angle of 35° to the line of centres (see diagram). The coefficient of restitution is 1. Find the speed and the direction of motion of B immediately after the collision. [8]

4



Two uniform rods AB and BC are freely jointed at B. The rods are in equilibrium in a vertical plane with AB freely jointed to a fixed point at A and BC freely jointed to a fixed point at C. The horizontal distance between A and B is 9 m and the horizontal distance between B and C is 7 m; the vertical distance between A and B is 12 m and the vertical distance between A and C is 1.5 m (see diagram). The weight of AB is 126 N and the weight of BC is 78 N. By taking moments about A for the rod AB and about C for the rod BC, find the magnitude of the force acting on AB at B. [8]

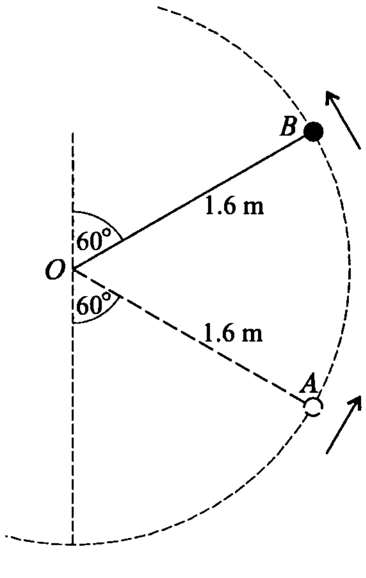
5 A car of mass 840 kg is accelerating on a straight horizontal road. It passes a point A with speed 10 m s^{-1} and when it has travelled a distance x metres beyond A its speed is v m s^{-1}. The engine develops a constant power of 26 kW and resistances are modelled by a force of $0.4v^2$ newtons opposing the motion.

(i) Show that $\dfrac{2100v^2}{65\,000 - v^3}\dfrac{dv}{dx} = 1$. [4]

(ii) Find the speed of the car when it has travelled 350 m beyond A. [5]

[Questions 6 and 7 are printed overleaf.]

6



A particle of mass 0.3 kg is attached to one end of a light inextensible string of length 1.6 m. The other end of the string is attached to a fixed point O. The particle moves in part of a vertical circle with centre O and radius 1.6 m. It passes through the point A where OA makes an angle of 60° with the downward vertical. The string becomes slack when the particle reaches the point B where OB makes an angle of 60° with the upward vertical (see diagram). Air resistance may be neglected.

(i) Find the speed of the particle when it is at B. [3]

(ii) Find the radial component of the acceleration of the particle when it is at A. [4]

(iii) Find the tension in the string when the particle is at A. [3]

7 A bungee jumper of mass 90 kg is attached to one end of a light elastic rope, with natural length 20 m and modulus of elasticity 1960 N. The other end of the rope is attached to a fixed point O. The jumper starts at rest at O and falls vertically. Air resistance may be neglected. When the rope has become taut and the extension of the rope is x metres, the jumper is moving downwards with speed $v\,\mathrm{m\,s^{-1}}$ and acceleration $a\,\mathrm{m\,s^{-2}}$.

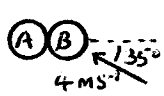
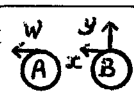
(i) Express a in terms of x, and verify that $a = 0$ when $x = 9$. [3]

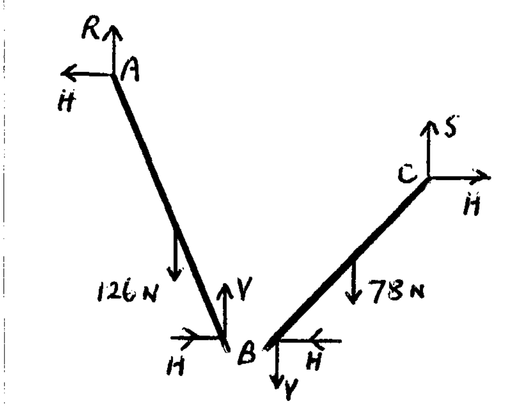
(ii) By considering energy, show that $45v^2 + 49x^2 = 882x + 17\,640$. [4]

(iii) Find the maximum speed of the jumper. [2]

(iv) Find the maximum extension of the rope. [3]

(v) Find the maximum magnitude of the acceleration of the jumper. [2]

1 (i)	$0.7 \times 12 \sin 30$ $= 4.2\ \text{m s}^{-1}$	M1 A1 **2**	*SR* Treat $0.7 \times 12 \cos 30$ as a MR (unless there is evidence to contrary)
(ii)	$I = 0.15 \times 4.2 - (-0.15 \times 6)$ $= 1.53\ \text{N s}$	M1 A1 **2**	
2 (i)	$-(m)g \sin\theta = (m)\left(2.45\dfrac{d^2\theta}{dt^2}\right)$ $\dfrac{d^2\theta}{dt^2} = -4\sin\theta \approx -4\theta$ Hence motion is approx SHM Period is $\dfrac{2\pi}{\sqrt{4}} = \pi = 3.14\ \text{s}$	B1 M1 A1 B1 **4**	Accept l or r instead of 2.45 For $\sin\theta \approx \theta$ Accept π
(ii)	$\theta = 0.08 \sin 2t$ When $t = 1.2$, $\theta = 0.08 \sin 2.4$ $= 0.054$ rad	B1 ft M1 A1 **3**	
3	Before:  Aft  $y = 4\sin 35 \quad (= 2.294)$ $0.6x + 0.3w = 0.6(4\cos 35)$ $w - x = 4\cos 35$ $x = \frac{4}{3}\cos 35 = 1.092$ Speed is $\sqrt{x^2 + y^2} = 2.54\ \text{m s}^{-1}$ Angle $\tan^{-1}\dfrac{y}{x} = 64.5°$ to line of centres	B1 M1 A1 B1 M1 M1 A1 A1 **8**	 Momentum equation or correct energy equation Obtaining a value of x Using $\sqrt{x^2 + y^2}$ or $\tan^{-1}\dfrac{y}{x}$

4	 Moments about A for AB $9V + 12H - 126 \times 4.5 = 0$ Moments about C for BC $7V - 10.5H + 78 \times 3.5 = 0$ $H = 36, \quad V = 15$ Magnitude is $\sqrt{H^2 + V^2} = 39$ N	M1 A1 M1 A1 M1 A1 M1A1 **8**	Moments equation Moments equation Obtaining H or V Both magnitudes correct *Solutions by other methods can earn full marks*
5 (i)	Driving force is $\frac{26000}{v}$ $\frac{26000}{v} - 0.4v^2 = 840v\frac{dv}{dx}$ $65000 - v^3 = 2100v^2\frac{dv}{dx}$ $\frac{2100v^2}{65000 - v^3}\frac{dv}{dx} = 1$	M1 M1 A1 A1 (ag) **4**	 Using N2L to obtain a diff eqn
(ii)	$x = \int \frac{2100v^2}{65000 - v^3}\,dv$ $= -700\ln(65000 - v^3) + C$ $v = 10$ when $x = 0 \Rightarrow C = 700\ln 64000$ $x = -700\ln(65000 - v^3) + 700\ln 64000$ When $x = 350$, $\ln(65000 - v^3) = 10.567$ $65000 - v^3 = 38820$ $v = 29.7\ \mathrm{m\,s^{-1}}$	M1 M1 A1 M1 A1 **5**	 or correct use of limits exponentiation
6 (i)	$T = 0, \quad 0.3 \times 9.8\cos 60 = 0.3 \times \frac{u^2}{1.6}$ $u = 2.8\ \mathrm{m\,s^{-1}}$	M1 A1 A1 **3**	N2L in radial direction

(ii)	By conservation of energy, $\frac{1}{2}(0.3)(v^2 - u^2) = 0.3 \times 9.8 \times 1.6$ $v^2 = 39.2$ Radial component of acceleration is $\frac{v^2}{1.6}$ $= 24.5 \text{ m s}^{-2}$	M1 A1 M1 A1 **4**	*Dependent on previous M1* M0 if mass included
(iii)	$T - 0.3 \times 9.8 \cos 60 = 0.3 \times 24.5$ $T = 8.82 \text{ N}$	M1A1 ft A1 **3**	Must use a result from (ii) A1ft requires numerical substitution
7 (i)	$T = \frac{1960}{20}x \quad (= 98x)$ $90 \times 9.8 - T = 90a$ $a = 9.8 - \frac{98}{90}x$ When $x = 9,\quad a = 9.8 - 9.8 = 0$	M1 A1 A1 (ag) **3**	
(ii)	Gain in EE is $\frac{1960x^2}{2 \times 20} = 49x^2$ Loss of PE is $90 \times 9.8(x + 20) = 882x + 17640$ By conservation of energy, $\frac{1}{2}(90)v^2 + 49x^2 = 882x + 17640$ $45v^2 + 49x^2 = 882x + 17640$	B1 B1 M1 A1 (ag) **4**	Equation involving KE, EE and PE
(iii)	Maximum speed when $a = 0$, i.e. $x = 9$ $v = 21.9 \text{ m s}^{-1}$	M1 A1 **2**	
(iv)	Maximum extension when $v = 0$ $49x^2 = 882x + 17640$ $x^2 - 18x - 360 = 0$ $(x - 30)(x + 12) = 0$ $x = 30 \text{ m}$	M1 M1 A1 **3**	Solving to obtain a value of x
(v)	Maximum $\lvert a \rvert$ when $x = 30$ $\lvert a \rvert = 22.9 \text{ m s}^{-2}$	M1 A1 **2**	Condone -22.9